Sciencia Acta Xaveriana
An International Science Journal
ISSN. 0976-1152

Volume 5
No. 2
pp. 39-38
September 2014

Super Vertex Mean Graphs

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Abstract : An edge labeling of a graph G is an assignment *f* of labels to the edges of G that induces a label for each vertex υ depending on the edges labels. A super vertex mean labeling *f* is an injection from E to the set {1,2,3,...,p+q} that induces for each vertex υ the label *Round* $\left(\frac{\sum_{e \in E_v} f(e)}{d(v)}\right)$ such that the set of all edge labels and the induced vertex labels is {1,2,3,...,p+q}. In this paper we study super vertex behavior of certain classes of graphs.

1. Introduction

A vertex labeling[1] of a graph G is an assignment *f* of labels to the vertices of G that induces a label for each edge uv depending on the vertex labels. An edge labeling of a graph G is assignment *f* of labels to the edges of G that induces a label for each vertex v depending on the edge labels. Let G = (V, E) be a simple graph with p vertices and q edges. A mean labeling *f* is an injection from V to the set (0,1,2,…, q) that induces for each edge uv the label $\left[\frac{\int(u)+\int(v)}{2}\right]$ such that the set of edge labels is (1, 2,…, q). Mean labeling was introduced by Somasundaram and ponraj [8]. A graph that accepts a mean labeling is known as mean graph. A super mean labeling *f* is an injection from V to the set {1,2,…, p+q} that induces for each edge uv the label $\left[\frac{\int(u)+\int(v)}{2}\right]$ such that the set of all vertex labels and the induced edge labels is

(1,2,…,p+q}. In this paper we study super vertex behavior of certain classes of graph. Super vertex mean labeling was introduced by R.Ponraj et al.[7]. A graph that accepts a super mean labeling is known as super mean graph. Some results on mean labeling and super mean labeling are given in [4, 5, 6, 7, 8, 9]. For a summary on various graph labeling see the Dynamic survey of graph labeling by Gallian [2]. Lourdusamy and Seenivasan [3] introduced vertex mean labeling as an edge analogue of mean labeling as follows: A vertex mean labeling of a *(p,q)* graph G(V,E) is defined as an injection f:E → {0,1,... , q^*}, $q^* = \max(p,q)$ such that the injection f: V → ℕ defined by the rule $f^v(V) = Round\left[\frac{\sum ev\int(e)}{d(v)}\right]$ satisfies the property that $f^v(V) = \{ f^v(u) : u \in V \} = \{1,2,..., p\}$, where E_v denotes the set of edges in G that are incident at v and ℕ denotes the set of all natural numbers. A graph that has a vertex mean labeling is called a vertex mean graph or V-mean graph. For all terminology and notations in graph theory, we refer the reader to the text book by D.B.West [10]. All graphs considered in this paper are finite and simple. Motivated by the concept of super mean labeling, we introduce super vertex mean labeling of graphs as follows

Definition 1.1. A Super vertex mean labeling f of a graph G(V,E) is an injection from E to the set {1,2,3,...,p+q} that induces for each vertex v the label $f^v(v)$ = Round $\left(\frac{\sum EV\int(e)}{d(v)}\right)$ such that the set of all edge labels and the induced vertex labels is {1,2,3,... , p+q}.

Henceforth we call super vertex mean as SVM. To initiate the investigation we obtain certain classes of graphs which are SVM graphs. It is obvious that no tree is an SVM graph. We also observe that C_4 is not an SVM graph.

1. Some SVM Graphs

Theorem 2.1. *The cycle C_n is an SVM graph if and only if n ? 4.*

Proof: Let $\{e_1, e_2, ..., e_n\}$ be the edge set of C_n such that $e_i = v_iv_{i+1}$, $1 \leq i \leq n-1$, $e_n = v_nv_1$. Checking each of the possibilities reveals that the cycle C_4 is not an SVM. So we assume that $n \neq 4$.

Case 1: $n \equiv 1 \pmod 2$. Let n = 2r+1. The edges of C_n are labeled as follows:

$$f(e_i) = \begin{cases} 2i-1 & if\ 1 \leq i \leq r+1 \\ 2i & if\ r+2 \leq i \leq n \end{cases}$$

It is easy to observe that f is injective. The induced vertex labels are given as follows

$$f^v(v_i) = \begin{cases} n+1 & if\ i = 1 \\ 2i-2 & if\ 2 \leq i \leq r+1 \\ 2i-1 & if\ r+2 \leq i \leq n \end{cases}$$

It is clear that $f(E) \cup f^v(V) = \{2i-1: 1 \leq i \leq n\} \cup \{2i: 1 \leq i \leq n\}$

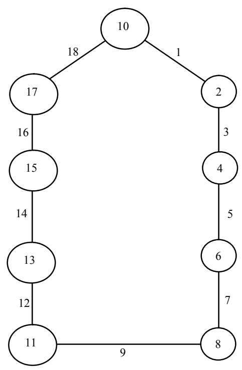
$= \{1,2,3, ..., 2n\}$.

Case 2: $n \equiv (mod\ 2)$. let n = 2r. The edges of C_n are labeled as follows:

$$f(ei) = \begin{cases} 1 & if\ i = 2 \\ 3 & if\ i = 2 \\ 7 & if\ i = 3 \\ 4i-4 & if\ 4 \leq i \leq r+1 \\ 4n-4i+5 & if\ r+2 \leq i \leq n-1 \\ 6 & if\ i = n \end{cases}$$

Hence the theorem.

Super vertex-mean labeling of C_9 and c_{10} are shown in Figure 1.



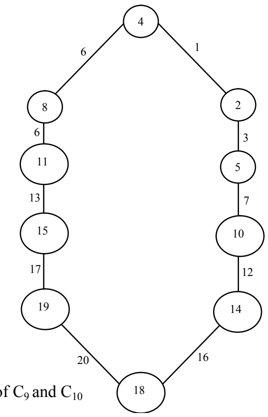
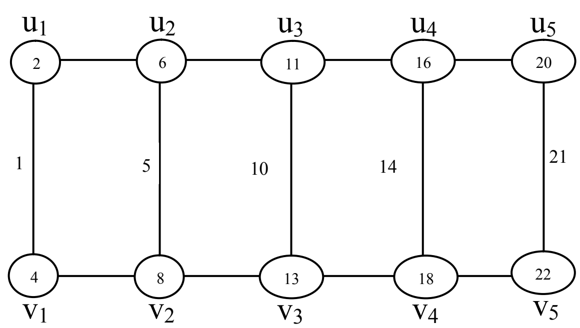


FIGURE 1. Super vertex-mean labeling of C_9 and C_{10}

Theorem 2.2

Prof. Let $V(P_n x P_2) = \{u_1, u_2, ..., u_n\}$ U $\{V_1.....v_2, ...vn)$ and $E (P_n \text{ x } P_2) = \{U_iU_{i+1} : 1 \leq i \leq n\text{-}1\}$ U $\{V_iV_{i+1} : \leq\ i \leq n\text{-}1\}$ U $\{u_iv_i : 1 \leq I < n\}$. We note that the order of L_n is 2n and the size is 3n-2



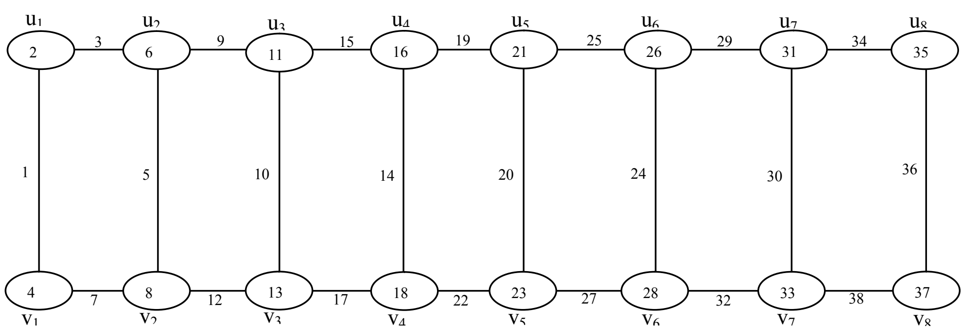


FIGURE 2. Super vertex-mean labeling of L_5

The edges of Ln are labeled as follows :

$$f(u_iu_{i+1}) = \begin{cases} 3 & \text{if } i = 1 \\ 5i-1 & \text{if } i \text{ is even or } i = n-1 \\ 5i & \text{if } i \text{ is odd and } i \neq 1 \text{ and } i \neq n-1 \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} 5n-2 & \text{if } i = n-1 \\ 5i+2 & \text{otherwise} \end{cases}$$

$$f(u_iv_i) = \begin{cases} 1 & \text{if } i = 1 \\ 5 & \text{if } i = 2 \\ 5i-6 & \text{if } i \text{ is even and } i \neq 2 \text{ or } n \\ 5i-5 & \text{if } i \text{ is odd and } i \neq 1 \text{ or } n \\ 5n-4 & \text{if } i = n \end{cases}$$

it is easily observed that f is injective. The induced vertex labels are as follows:

$$f^v(u_i) = \begin{cases} 2 & \text{if } i = 1 \\ 5i-4 & \text{if } 2 \leq i \leq n-1 \\ 5n-5 & \text{if } i = n \end{cases}$$

$$f^v(v_i) = \begin{cases} 4 & \text{if } i = 1 \\ 5i-2 & \text{if } 2 \leq i \leq n-1 \\ 5n-3 & \text{if } i = n \end{cases}$$

It is easy to verify, in both cases, that the set of all edge labels and the induced vertex labels is {1,2,....,5n-2}.

Hence the theorem.

SVM labeling of L_5 and L_8 are shown in Figure 2.

Theorem 2.3. *A triangular snake with n blocks is SVM.*

Proof. Let G_n be a triangular snake with n blocks on p vertices and q edges. Then $p = 2n + 1$ and $q = 3n$.

Let $V(G_n) = \{u_i : 1 \leq i \leq n+1\} \cup \{v_i : 1 \leq i \leq n\}$ and $V(G_n) = \{u_iu_{i+1}, u_iv_i, u_{i+1}v_i : 1 \leq i \leq n\}$. The order of G_n is 2n+1 and size is 3n.

The edges of G_n are labeled as follows :

$$f(u_iu_{i+1}) = \begin{cases} 1 & \text{if } i = 1 \\ 5i & \text{if } i \text{ is even and } i \neq n \\ 5i-3 & \text{if } i \text{ is odd and } i \neq 1 \\ 5n+1 & \text{if } n \text{ is even and } i = n \end{cases}$$

$$f(u_iv_i) = \begin{cases} 5i-3 & \text{if i is even} \\ 5i-2 & \text{if i is odd} \end{cases}$$

$$f(v_iu_{i+1}) = \begin{cases} 5i-1 & \text{if i is even} \\ 5i & \text{if i is odd and } i \neq n \\ 5n+1 & \text{if n is odd and } i = n \end{cases}$$

Then, the induced vertex labels are as follows:

$$f^v(u_i) = \begin{cases} 2 & if\ i = 1 \\ 5i-4 & if\ 2 \leq i \leq n \\ 5n & if\ i = n+1\ and\ n\ is\ even \\ 5i-1 & if\ i = n+1\ and\ n\ is\ odd \end{cases}$$

$$f^v(v_i) = \begin{cases} 5i-2 & if\ i\ is\ even \\ 5i-1 & if\ i\ is\ odd\ and\ i \neq n \\ 5n & if\ i = n\ and\ n\ is\ odd \end{cases}$$

It can be easily verified that f is injective and the set of edge labels and induced vertex labels is {1,2,...., 5n+1}.

Hence the theorem.

Super vertex mean labeling of triangular snakes are shown in Figure 3.

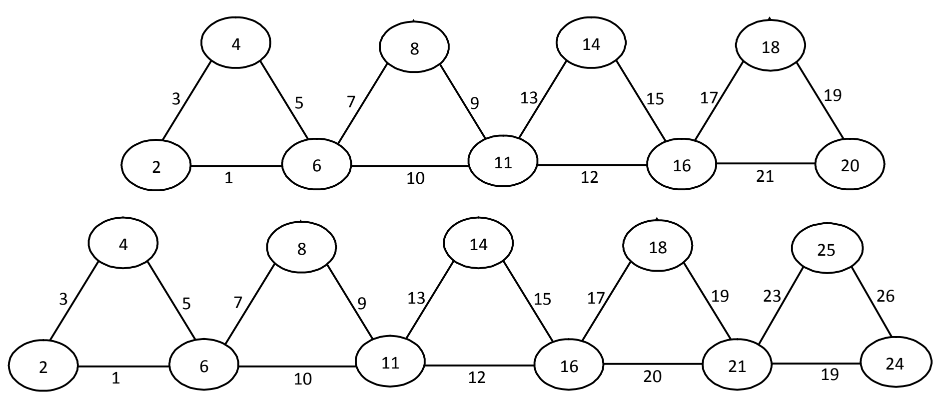


Figure 3 : Super vertex-mean labeling of triangular snakes

Theorem 2.4. *Quardrilateral snakes are SVM.*

Proof. Let G_n be a quadrilateral snake with $V(G_n) = \{u_i : 1 \le i \le n+1\} \cup \{u_i, w_i : 1 \le i \le n\}$ and $E(G_n) = \{u_iu_{i+1}, u_iv_i, u_{i+1}w_i, v_iw_i : 1 \le i \le n\}$. Then $p = 3n + 1$ and $q = 4n$. Define f: $E(G_n) \rightarrow \{1,2,3,..., 7n+1\}$ as follows:

$$f(u_iu_{i+1}) = \begin{cases} 7i & \text{if } 1 \le i \le n-1 \\ 7n+1 & \text{if } i = n \end{cases}$$

$$f(u_iv_i) = 7i-6 \quad \text{if } 1 \le i \le n$$

$$f(v_iw_i) = \begin{cases} 3 & \text{if } i = 1 \\ 7i-3 & \text{if } 2 \le i \le n \end{cases}$$

$$f(w_i\, u_{i+1}) = 7i-1 \quad \text{if } 1 \le i \le n$$

Then the induced vertex labels are as follows :

$$f^v(u_i) = \begin{cases} 4 & \text{if } i = 1 \\ 7n & \text{if } i = n+1 \\ 7i-5 & \text{otherwise} \end{cases}$$

$$f^v(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 7i-5 & \text{otherwise} \end{cases}$$

$$f^v(w_i) = 7i-2 \quad \text{if } 1 \le i \le n$$

It can be easily verified that f is injective and the set of edge labels and induced vertex labels I {1,2,3,..., 7n+1}.

Hence the theorem.

A Super vertex-mean labeling of a Quadrilateral snake is shown in Figure 4.

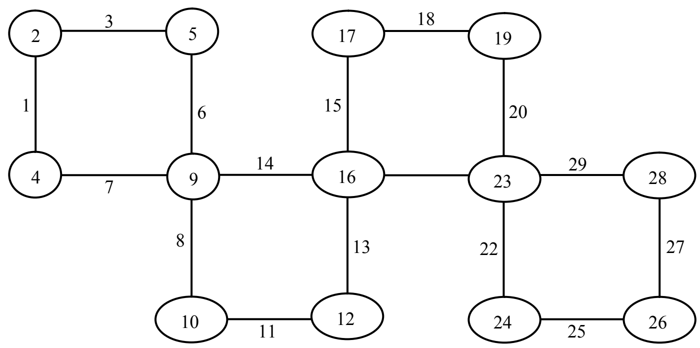


FIGURE 4. A Super vertex-mean labeling of a Quadrilateral snake

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